

Philosophy 324A
Philosophy of Logic
2016

Note Seven

SOME REMARKS ON TARSKI ON TRUTH

1. *Soundness and completeness*

Alfred Tarski (1901-1983), along with Abraham Robinson (1918-1974), is regarded as the principal founder of modern model theory. As we have seen, one of model theory's chief objectives is to find a formal representation of the natural language predicate "is true", whereby for each true sentence of a natural language theory there would be an assured formal representation of it in the formal language of the model theory of a logistic system. Similarly for logical truth. For any logical truth of a theory formulated in a natural language there would be a unique formal sentence of the model A such that $\models A$; that is to say, A has a model in every possible interpretation of the system. One of the great achievements of classical model theory (which was also a boon to modern proof theory) is that for every formal sentence A such that $\models A$ there is a formal sentence A of proof theory such that $\vdash A$. In other words, classical logic (and many nonclassical ones too) are both *sound* and *complete*. For every formal representation of a logically true English sentence there is a unique representation of a provable English sentence, the same sentence in each case. It holds the other way around too. For every formal representation of a provable sentence of English there is a unique formal representation of a logically true English sentence, again the same sentence in each case.

2. *Paradox*

When he was working out the mechanics of model theory, Tarski can hardly have been unaware of the Liar Paradox, which appears to have originated with Epimenides of Crete (*fluorit* C. 600 BC) who is believed to have said "All Cretans are liars". Of course, this is more a joke than a paradox. We don't get a paradox unless we construe him as having said "Anything any Cretan says is untrue, and I who say this am a Cretan." One of the common examples leaves the reputation of Cretans unmolested. The version is called the Strengthened Liar. (To get the plain old Liar, just substitute "false" for each occurrence of "untrue" and "not true".

- (1) (1) is not true
- (2) Sentence (1) is either true or not. (LEM)
- (3) If (1) is true, then what it says is so, namely, that it's not true.
- (4) If (1) is not true then it is the way that (1) says it is, hence is not true.
- (5) So (1) is true iff it is not.
- (6) Hence (1) is true and (1) is not true.

On the face of it, this is a proof by contradiction or a *reductio* proof of at least one of the lines preceding (5) or, if not that, of the invalidity of the derivation. Hardly anyone is much inclined to play the invalidity card (although some actually have done so), while many favour pinning the wrap on line one.

- Some people think that the problem with (1) is self-reference. However, there seems to be nothing wrong with self-reference as such: “In saying the very words that I’m uttering right now, I am saying something in English” seems unarguably true, indeed a demonstration of its truth.

Others think that sentence on line one is defectively structured. It is hard to see why. The sentence to the right of the first occurrence of the parenthesized expression “(1)” is a perfectly grammatical and well-formed sentence of English which via its own occurrence of “(1)”, achieves anaphoric reference to the parenthesized numeral that refers to it.

If it is true that there are fifty shades of grey, then perhaps sentences such “John has grey hair” might well be not false without ever being true. Even if we say that there are fifty different truth values only one of which is what that sentence has, the fact remains that it could be either of forty-nine of those fifty without being untrue. (That’s one of the reasons for presenting the puzzle posed by the Liar in the strengthened version.)

Here is an interesting passage from Simon Blackburn’s *Oxford Dictionary of Philosophy*, 2nd paperback edition, Oxford: Oxford University Press, 2005, in the entry on Tarski at pp. 360-361:

Tarski ... “moved to the University of California at Berkeley in 1942. Together with Abraham Robinson ... he created the mathematical theory of models, and his programme at Berkeley and indefatigable energy was responsible for spreading mathematical logic and model theory worldwide. He worked on decidable and undecidable axiomatic systems, and in the course of his mathematical career he published over 300 papers and books, on topics ranging from set theory to geometry and algebra. Tarski’s logical work is well seen in *A Decision Method for Elementary Algebra and Geometry* (1948), and for philosophers he is best represented in the collection *Logic, Semantics, and Metamathematics* (1956).¹

It is interesting that there is no reference in this entry to Tarski’s work on the Liar, beyond a veiled reference to the papers on this subject contained in the book “for philosophers”. I infer from this that Tarski’s “The concept of truth in formalized languages”, which appeared in German in 1935 and the year before in Polish, is not considered by Blackburn to be a contribution to mathematical logic or model theory, but is rather an essay on the theory of truth for *natural languages*. (Hereafter, I’ll relativize this to English.)

3. Regimentation

If we now turn to Beall and Restall’s chapter 3, we’ll come upon a number of interesting ideas to which we’ll have occasion from time to time to return. A bit later, they say something quite specific about the observation of a sentence ago:

We define *truth-in-a-model* for sentences of *English* by the standard processes of regimentation of those sentences, thereby achieving an account of *formal validity* for natural language arguments. Call that account the *Tarskian account of validity of*

¹ A second edition, edited by John Corcoran, appeared with Hackett in 1983.

arguments in natural language ('TM', for Tarskian models). The emphasis in the first instance is mine.²

For a loose indication of what a *regimented sentence* might be, see p. 9 of *Logical Pluralism*. The name originates with Quine. To regiment a chunk of English from the perspective of formal languages, two things would be required:

- We will have to adjust the formatting of English sentences to minimize syntactic ambiguity (e.g. by using parentheses) and to clarify underlying grammatical form (e.g. replacing “All men are mortal” with “For anything at all, if it is a man then it is mortal”).
- We would also enrich the lexicon of English by adding the logical particles of logic as neologisms. For example, replace “All men are mortal” with “ $\forall x (\text{man } (x) \supset \text{mortal } (x))$.”

Getting back now to the Tarski (!) paradox – i.e. the Liar – it is essential to distinguish Tarski’s two responses to it. The first was like Frege’s response to the Russell paradox. The second was more like Russell’s response to it. Before examining this difference, it is helpful to see that by “the concept of truth in formalized languages”,

- Tarski does *not* mean formal languages of logistic systems.
- He *does* mean is natural languages *regimented* in the manner of Quine (although he pre-dates Quine on this point).

The point to hold onto before going to his reactions to the Liar is that Tarski is intent on providing for the English predicate “is true” nothing less than a *rational reconstruction* of it using the formal semantics of formal languages as a *template*. In this way, Tarski uses the *stipulated* properties of model theory as a way of clarifying the corresponding properties of English. This, one way of rationally reconstructing the intuitive of truth is by having the formally stipulated true sentence of model theory formally represent them.

What we have here is yet another example of unannounced changes of meaning. When Tarski first was talking about formal semantics he meant “model theory”. When, in the 1935 paper, he started talking about a formalized semantics for English, he meant a theory of the intuitive property truth as applied to sentences of English *regimented* in accordance with certain aspects of the model theoretic template.

A final quick word about regimentation. As far as I know, this usage of the word in arose with Quine, although when he speaks about a regimented language, he prefers saying that they are natural languages (or significant fragments of them) in *canonical notation*. Jaakko Hintikka is helpful here. He writes

² I persist in the claim that “argument” is not *le mot juste* here. “Deduction”, in the sense of sequences of sentences whose terminal sentences are deductively derivable from the preceding ones, is the better choice. In Tarski’s *Introduction to Logic and to the Methodology of Deductive Sciences*, published in Polish in 1936 – one year after the concept of truth paper – and first appeared in English in 1941 with the Oxford University Press, there is *no* reference to arguments in the index beyond those that are inputs to functions. However, the broader use of “argument” is now so deeply dug-in among logicians that we’d better just bite our tongues and go with the flow.

It is admittedly true that the connections between logicians' canonical notations and our familiar vernacular ... play an extremely important role in developing the theory of formal languages, and especially their semantics. But such connections are best viewed as happy outcomes of applications of one's basic logical semantic theory, which must be developed first. Such pleasant connections ... are hoped-for end products of formal semantics, not its starting-points.³

Here's what I take Hintikka to be saying. Whether model theoretic entities and methods do any philosophical good in relation to natural language properties of interest is all down to the adequacy of the *representation relations* the former bear to the latter. Representational adequacy is not simply to be assumed. It must be shown.⁴

4. Tarski's reaction to the Liar

In "The concept of truth in formalized languages" and the later "The semantic conceptions of truth"⁵, Tarski reacts to the paradox in two quite different and apparently irreconcilable ways:

Tarski I: The paradox of the Liar demonstrated the fatal inconsistency of the intuitive concept of truth, implying thereby that the English predicate "is true" has a null extension, and therefore that no statement of natural language is true (or false either). (Note the similarity of Tarski I to Frege's response to the paradox of sets.) Since the paradox that demolishes truth is a validly derived consequence of the intuitive theory of truth, intuitive semantics suffers an irrecoverable collapse. Since a truth predicate is necessary for the workings of natural languages, the contagion of the Liar also brings English and her sister languages permanently down. (Recall Frege's early claim that, owing to vagueness and ambiguity, no statement of a natural language can be either true or false.)

Tarski II: The Liar presents intuitive semantics with challenges that closely resemble the challenges to intuitive mathematics brought about by the Russell paradox. They will have to be met in the way that Russell and others dealt with the set theoretic ones. The intuitive concept of truth is unusable in its present condition. The same is true for intuitive semantics. Each of these concepts is in need of urgent repair, and their repair can be achieved – in JW's words, not Tarski's – by judicious application of the stipulation-formal representation – rational reconstruction dynamic. It was never in Russell's mind that the set theoretic paradox came anywhere close to putting intuitive mathematics out of

³ Jaakko Hintikka, "Quine on who's who", in Lewis Hahn and Paul Arthur Schilpp, editors, *The Philosophy of W. V. Quine*, expanded edition, volume 18 of their *Library of Living Philosophers*, pages 209-223, Chicago and LaSalle: Open Court 1998; pp. 212-213. First edition in 1986.

⁴ Hintikka goes on to issue this admonition: "Were it not for the semantical clarity of formal languages, a favorite strategy of many theorists of language would not make much sense. This is the strategy of elucidating the phenomena of natural languages by trying to translate their sentences into a formal logician's 'canonical notation'. For if the latter were not semantically superior to our informal jargons, what would be gained by such translations?" (*idem.*) By the way, don't forget that a *logician's* translation relation is the converse of his or her formal representation relation.

⁵ Reprinted in Herbert Feigl and Wilfrid Sellars, editors, *Readings in Philosophical Analysis*, pages 51-84, New York: Appleton-Century-Crofts, 1949.

business (Russell's logicism was never predicated on the belief that pre-paradox mathematics was unintelligible; and that remained his position even after the paradox was discovered. This is important. The logicist desire to re-express all of mathematics in pure logic has nothing to do with the trouble caused the paradox.)

Tarski I has had no lasting influence on formal approaches to analytic philosophy. Tarski II is a different matter entirely. Tarski II replicates how mathematicians have dealt with post-paradox mathematics. As already mentioned, ZF (ZFC) achieved an early prominence and is widely thought to be the best treatment of sets we've so far arrived at. (There are other worthy contenders, but we needn't go into this further for present purposes. However, we'll get back to it when we take on the multiplicity and strife problem.)

5. *Hierarchies*

In ZF and various other modern treatments, sets are characterized by tampering with the membership relation. In the old approach, set theoretic members are undistinguished as to kind. In the new ones, membership is distributed over transfinitely many different kinds of thing. Sets are ranked hierarchically by level. On this, the *iterative* conception, the first level contains the empty set \emptyset , and by forming different levels of sets of sets, the whole of the set theoretic hierarchy is generated. One virtue of this approach is that no set on any level can be a member of itself.

Tarski II behaves in like manner. Instead of the all-purpose solitary truth predicate of English, there would be a transfinite number of made-up ones, each arranged in an ascending hierarchy of different instantiation-levels. Accordingly, the newly reformed English predicate "is true" would be in common English usage transfinitely ambiguous. The intuitive semantics for English would be up-graded accordingly, once English was put into canonical notation.

Of course, there isn't the slightest empirical evidence that the English word "true" has infinitely many different meanings (and ditto for "set"). So the next move is to say that the ambiguities of "true" apply only to the "deep structure" of English in which they lie safely out of the way from encumbering its day-to-day communicational fluency. (Gosh, who knew?)